Rešiti integrale

1. $\int (4x^2 + 2x - 3)dx$

rešenje:

$$\int (4x^2 + 2x - 3)dx =$$

$$= 4\int x^2 dx + 2\int x dx - 3\int dx = 4 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} - 3x + C$$

2. $\int \left(\frac{1}{x^2} + \frac{2}{x^3} + \frac{1}{4x^4} \right) dx$

rešenje:

$$\int \left(\frac{1}{x^2} + \frac{2}{x^3} + \frac{1}{4x^4} \right) dx =$$

$$= \int x^{-2} dx + 2\int x^{-3} dx + \frac{1}{4}\int x^{-4} dx = \frac{x^{-2+1}}{-2+1} + 2 \cdot \frac{x^{-3+1}}{-3+1} + \frac{1}{4} \cdot \frac{x^{-4+1}}{-4+1} + C =$$

$$= -\frac{1}{x} - \frac{1}{x^2} - \frac{1}{12} \cdot \frac{1}{x^3} + C$$

3. $\int \left(\sqrt[3]{x} + \frac{1}{\sqrt{x}} \right) dx$

rešenje:

$$\int \left(\sqrt[3]{x} + \frac{1}{\sqrt{x}} \right) dx = \int x^{\frac{1}{3}} dx + \int x^{-\frac{1}{2}} dx =$$

$$= \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{3}{4} \cdot x^{\frac{4}{3}} + 2 \cdot x^{\frac{1}{2}} + C$$

4. $\int \frac{(2x+3)^2}{x^2} dx$

rešenje:

$$\int \frac{(2x+3)^2}{x^2} dx = \int \frac{4x^2 + 12x + 9}{x^2} dx =$$

$$= \int \frac{4x^2}{x^2} dx + \int \frac{12x}{x^2} dx + \int \frac{9}{x^2} dx =$$

$$= 4\int dx + 12\int \frac{dx}{x} + 9\int \frac{1}{x^2} dx = 4x + 12\ln|x| - 9 \cdot \frac{1}{x} + C$$

Rešiti integrale (Integracija metodom smene)

5. $\int (2x+3)^6 dx$

rešenje:

$$smena: \quad 2x + 3 = t \quad \Rightarrow \quad dx = \frac{dt}{2}$$

$$\int t^6 \frac{dt}{2} = \frac{1}{2}\int t^6 dt = \frac{1}{2} \cdot \frac{t^7}{7} + C = \frac{1}{14} \cdot (2x+3)^7 + C$$

6. $\int \frac{dx}{1-3x}$

rešenje:

$$smena: \quad 1-3x=t \quad \Rightarrow \quad dx=\frac{dt}{-3}$$

$$\int \frac{dt}{-3t}=-\frac{1}{3}\int \frac{dt}{t}=-\frac{1}{3}\ln|t|+C=-\frac{1}{3}\ln|1-3x|+C$$

7. $\int \frac{x}{1+x^2}dx$

$$smena: \quad 1+x^2=t \quad \Rightarrow \quad xdx=\frac{dt}{2}$$

$$\int \frac{1}{t}\cdot\frac{dt}{2}=\frac{1}{2}\int \frac{dt}{t}=\frac{1}{2}\ln|t|+C=$$

$$=\frac{1}{2}\ln\left(1+x^2\right)+C=\ln\sqrt{1+x^2}+C$$

8. $\int xe^{x^2}dx$

rešenje:

$$smena: \quad x^2=t \quad \Rightarrow \quad xdx=\frac{dt}{2} \quad \Rightarrow$$

$$\int e^t\cdot\frac{dt}{2}=\frac{1}{2}\int e^t dt=\frac{1}{2}e^t+C=\frac{1}{2}e^{x^2}+C$$

9. $\int \frac{(\ln x)^2}{x}dx$

rešenje:

$$smena: \quad \ln x=t \quad \Rightarrow \quad \frac{dx}{x}=dt \quad \Rightarrow$$

$$\int t^2 dt=\frac{t^3}{3}+C=\frac{\ln^3 x}{3}+C$$

10. $\int \frac{dx}{x\ln^2 x}$

rešenje:

$$smena: \quad \ln x=t \quad \Rightarrow \quad \frac{dx}{x}=dt \quad \Rightarrow$$

$$\int \frac{dt}{t^2}=\int t^{-2}dt=\frac{t^{-2+1}}{-2+1}+C=-\frac{1}{t}+C=-\frac{1}{\ln x}+C$$

11. $\int xe^{x^2+1}dx$

rešenje:

$$smena: \quad \Rightarrow \quad x^2+1=t \quad \Rightarrow \quad xdx=\frac{dt}{2} \quad \Rightarrow$$

$$\int e^t\cdot\frac{dt}{2}=\frac{1}{2}\int e^t dt=\frac{1}{2}e^t+C=\frac{1}{2}e^{x^2+1}+C$$

12. $\int \frac{dx}{\sin x}$

rešenje:

$$\int \frac{dx}{\sin x} = \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \frac{1}{2}\int \frac{dx}{\sin\frac{x}{2}\cos\frac{x}{2}} =$$

$$= \frac{1}{2}\int \frac{dx}{\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\cdot\cos^2\frac{x}{2}} = \frac{1}{2}\int \frac{dx}{tg\frac{x}{2}\cos^2\frac{x}{2}} =$$

$$= \int \frac{dt}{t} = \ln|t| + C = \ln\left|tg\frac{x}{2}\right| + C$$

$$smena: \quad tg\frac{x}{2} = t \quad \Rightarrow \quad \frac{dx}{2\cos^2\frac{x}{2}} = dt$$

13. $\int \frac{dx}{\cos x}$

rešenje:

$$\cos x = \sin\left(\frac{\pi}{2} + x\right) \quad \Rightarrow$$

$$\int \frac{dx}{\cos x} = \int \frac{dx}{\sin\left(\frac{\pi}{2} + x\right)} = \ln\left|tg\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$$

$$smena: \quad \frac{\pi}{2} + x = t \quad dalje\ kao\ u\ prethodnom\ zadatku$$

14. $\int tg^2 x dx$

rešenje:

$$\int \frac{\sin^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx =$$

$$= \int \frac{dx}{\cos^2 x} - \int dx = tgx - x + C$$

15. $\int \frac{dx}{\sqrt{x+1}-\sqrt{x}}$

rešenje:

$$\int \frac{\sqrt{x+1}+\sqrt{x}}{\left(\sqrt{x+1}-\sqrt{x}\right)\cdot\left(\sqrt{x+1}+\sqrt{x}\right)}dx =$$

$$= \int\left(\sqrt{x+1}+\sqrt{x}\right)dx = \int\sqrt{x+1}dx + \int\sqrt{x}dx =$$

$$= \frac{(x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{2}{3}\cdot(x+1)^{\frac{3}{2}} + \frac{2}{3}\cdot x^{\frac{3}{2}} + C$$